Compuertas de un cubit - 3^a sesión

Computación Cuántica: Algorítmica y Software

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Algunas propiedades y notaciones

$$\langle u|\lambda v\rangle = \langle \lambda^* u|v\rangle = \lambda \langle u|v\rangle \qquad \langle u|v\rangle = \langle v|u\rangle^*$$

$$(AB)^\dagger = B^\dagger A^\dagger$$

$$\langle u|Av\rangle = \langle A^\dagger u|v\rangle \qquad [\ u^* A v = (u^* A) v = (A^* u)^* v\ ]$$

$$\langle u|A|v\rangle = \langle u|Av\rangle = \langle A^\dagger u|v\rangle$$

$$|\psi\rangle^\dagger = \langle\psi|$$ [esta notación no se utiliza]

Matrices unitarias

Definición: $UU^\dagger = I$ es decir $U^\dagger = U^{-1}$

$$UU^\dagger = I \implies \langle Uv|Uv\rangle = \langle U^\dagger Uv|v\rangle = \langle v|v\rangle$$

$$\implies \|Uv\| = \|v\|$$

$\|Uv\| = \|v\| \implies UU^\dagger = I$ [demostración como ejercicio]

las compuertas cuánticas son unitarias

Matrices unitarias de 1-qubit (1)

$$U = \begin{pmatrix} a & c \\ b & d \end{pmatrix} = \begin{pmatrix} u & v \end{pmatrix} \qquad u = \begin{pmatrix} a \\ b \end{pmatrix} \quad v = \begin{pmatrix} c \\ d \end{pmatrix}$$

$$U^\dagger = \begin{pmatrix} a^* & b^* \\ c^* & d^* \end{pmatrix} \qquad U^\dagger U = \begin{pmatrix} aa^* + bb^* & a^*c + b^*d \\ ac^* + bd^* & cc^* + dd^* \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{cases} aa^* + bb^* = 1 \\ cc^* + dd^* = 1 \\ a^*c + b^*d = \langle u|v\rangle = 0 \\ ac^* + bd^* = 0 \end{cases} \iff \begin{cases} \|u\| = 1 \\ \|v\| = 1 \\ a^*c = -b^*d \end{cases}$$

Matrices unitarias de 1-qubit (2)

$$U = \begin{pmatrix} a & c \\ b & d \end{pmatrix} = \begin{pmatrix} u & v \end{pmatrix} \qquad a^*c = -b^*d \qquad \|u\| = 1 \quad \|v\| = 1$$

si $a \neq 0$ tomamos $\lambda = \frac{d}{a^*}$ $\qquad c = -\lambda b^* \quad d = \lambda a^*$

[si $a = 0$ entonces $b \neq 0$ tomamos $\lambda = -\frac{c}{b^*}$] $\qquad U = \begin{pmatrix} a & -\lambda b^* \\ b & \lambda a^* \end{pmatrix}$

$$1 = |-\lambda b^*|^2 + |\lambda a^*|^2 = |\lambda|(|b^*|^2 + |a^*|^2) \implies |\lambda| = 1 \qquad \lambda = e^{2i\omega}$$

$$U = \begin{pmatrix} a & -e^{2i\omega}b^* \\ b & e^{2i\omega}a^* \end{pmatrix} = e^{i\omega}\begin{pmatrix} e^{-i\omega}a & -e^{i\omega}b^* \\ e^{-i\omega}b & e^{i\omega}a^* \end{pmatrix} = e^{i\omega}\begin{pmatrix} e^{-i\omega}a & -(e^{-i\omega}b)^* \\ e^{-i\omega}b & (e^{-i\omega}a)^* \end{pmatrix}$$

$$\alpha = e^{-i\omega}a \quad \beta = e^{-i\omega}b \qquad \boxed{U = e^{i\omega}\begin{pmatrix} \alpha & -\beta^* \\ \beta & \alpha^* \end{pmatrix}} \qquad |\alpha|^2 + |\beta|^2 = 1$$

Matrices unitarias especiales

$$U = \begin{pmatrix} \alpha & -\beta^* \\ \beta & \alpha^* \end{pmatrix} \qquad |\alpha|^2 + |\beta|^2 = 1$$

- Las matrices unitarias especiales forman un grupo para la multiplicación

$$R_x(\theta) = X_\theta = e^{-i\frac{\theta}{2}X} \qquad R_y(\theta) = Y_\theta = e^{-i\frac{\theta}{2}Y} \qquad R_z(\theta) = Z_\theta = e^{-i\frac{\theta}{2}Z}$$

- Las matrices de rotación son unitarias especiales

La compuerta universal Qiskit U3/UGate

$$U_3(\theta, \phi, \lambda) = \begin{pmatrix} \cos\frac{\theta}{2} & -e^{i\lambda}\sin\frac{\theta}{2} \\ e^{i\phi}\sin\frac{\theta}{2} & e^{i(\phi+\lambda)}\cos\frac{\theta}{2} \end{pmatrix}$$

$U_3(\theta, \phi, \lambda) \backsimeq Z_\phi Y_\theta Z_\lambda$ [descomposición ZYZ]

$U_3(\theta, \phi, \lambda) \backsimeq Z_\phi X^{-\frac{1}{2}} Z_\theta X^{\frac{1}{2}} Z_\lambda$ [compuertas nativas transmon]

Ejercicios

$Y_\theta \backsimeq U_3(\_, \_, \_)$

$X_\theta \backsimeq U_3(\_, \_, \_)$

$Z_\theta \backsimeq U_3(\_, \_, \_)$

$H \backsimeq U_3(\_, \_, \_)$

$X, Y, Z, X^{\frac{1}{2}}, X^{-\frac{1}{2}}, Y^{\frac{1}{2}}, Y^{-\frac{1}{2}}$