Ejercicio

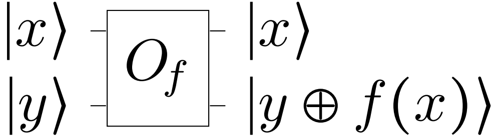
$f : \{0,1\} \to \{0,1\}$ constantes equilibradas

$$f_0 \begin{cases} 0 \mapsto 0 \\ 1 \mapsto 0 \end{cases} \qquad f_2 \begin{cases} 0 \mapsto 0 \\ 1 \mapsto 1 \end{cases}$$

$$f_1 \begin{cases} 0 \mapsto 1 \\ 1 \mapsto 1 \end{cases} \qquad f_3 \begin{cases} 0 \mapsto 1 \\ 1 \mapsto 0 \end{cases}$$

Escribir los circuitos y las matrices de los oráculos O_{f_0}, O_{f_1}, O_{f_2} y O_{f_3}

Oráculo para f



$x \in \{0,1\}^n \qquad y \in \{0,1\}$

O_f permuta vectores de la base

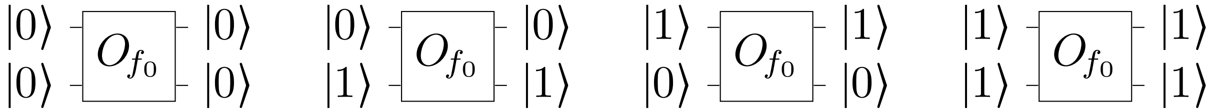
$0 \oplus 0 = 0 \qquad 1 \oplus 0 = 1 \qquad 0 \oplus 1 = 1 \qquad 1 \oplus 1 = 0 \qquad$ [$\oplus$: XOR]

$O_f\,|x,y\rangle = |x, y \oplus f(x)\rangle \qquad$ [Notación: $|x\rangle \otimes |y\rangle = |x\rangle\,|y\rangle = |x,y\rangle = |xy\rangle$]

$$O_f O_f\,|x,y\rangle = O_f\,|x, y \oplus f(x)\rangle = |x, y \oplus f(x) \oplus f(x)\rangle = |x, y \oplus 0\rangle = |x,y\rangle$$

$O_f = O_f^{-1} \qquad O_f$ es hermitiano (unitario y autoinversa)

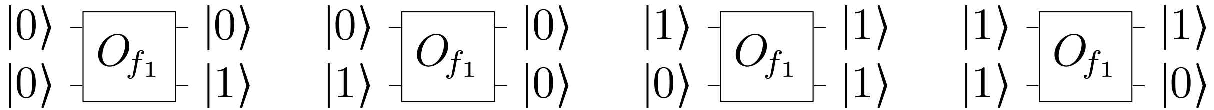
$$f_0(0) = 0 \quad f_0(1) = 0$$



$$O_{f_0} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

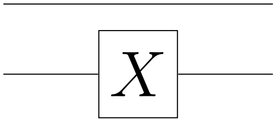
el circuito es:

$f_1(0) = 1 \quad f_1(1) = 1$

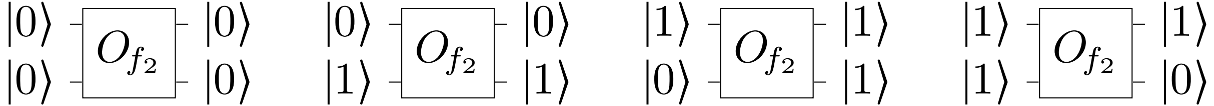


$$O_{f_1} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

el circuito es:

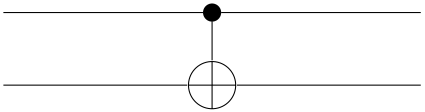


$f_2(0) = 0 \quad f_2(1) = 1$

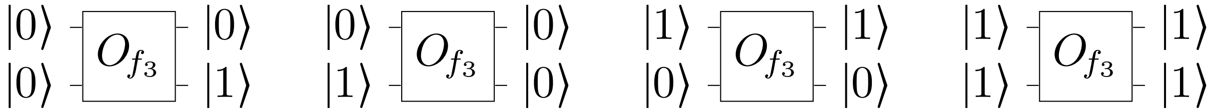


$$O_{f_2} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

el circuito es:



Ejemplo con f_3 $\quad f_3(0) = 1 \quad f_3(1) = 0 \quad$ [f_3: NOT]



$$O_{f_3}\,|00\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \qquad O_{f_3}\,|01\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad O_{f_3}\,|10\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \qquad O_{f_3}\,|11\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$O_{f_3} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad O_{f_3}\begin{pmatrix} \alpha \\ \beta \\ \gamma \\ \delta \end{pmatrix} = \begin{pmatrix} \beta \\ \alpha \\ \gamma \\ \delta \end{pmatrix}$$

el circuito para f_3 es: