

Compuertas de 2 cubits

Computación Cuántica: Algorítmica y Software

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2025 UCV

Estado cuántico de 2 cubits

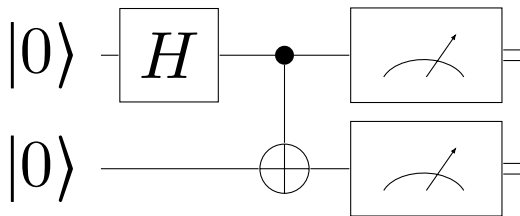
$$|00\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad |01\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad |10\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad |11\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

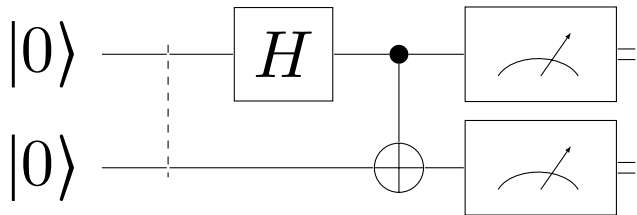
$$|\psi\rangle = \begin{pmatrix} \alpha \\ \beta \\ \gamma \\ \delta \end{pmatrix} = \alpha |00\rangle + \beta |01\rangle + \gamma |10\rangle + \delta |11\rangle$$

Circuitos

```
\documentclass{article}
\usepackage{yquant}
\usepackage{braket}

\begin{document}
  \begin{tikzpicture}[scale=2]
    \begin{yquant}
      qubit {\ket{0}} q[2];
      h q[0];
      cnot q[1] | q[0];
      measure q;
    \end{yquant}
  \end{tikzpicture}
\end{document}
```





$$|0\rangle \otimes |0\rangle = |0\rangle |0\rangle = |00\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} =$$

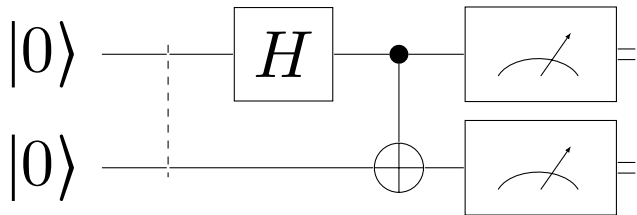
Producto de Kronecker (1)

Producto de Kronecker = producto tensorial para matrices

Matriz = tensor de rango 2 = dimensiones $n \times m$

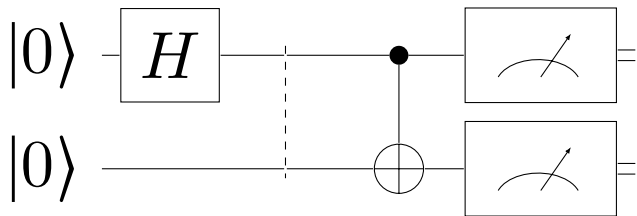
Vector (columna) = matriz de dimensiones $n \times 1$

$$\begin{pmatrix} a \\ b \end{pmatrix} \otimes \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} a \begin{pmatrix} c \\ d \end{pmatrix} \\ b \begin{pmatrix} c \\ d \end{pmatrix} \end{pmatrix} = \begin{pmatrix} ac \\ ad \\ bc \\ bd \end{pmatrix}$$



$$|0\rangle \otimes |0\rangle = |0\rangle |0\rangle = |00\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} =$$

$$\begin{pmatrix} 1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ 0 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$



$$H |0\rangle \otimes |0\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$$

$$H |0\rangle \otimes I |0\rangle = (H \otimes I)(|0\rangle \otimes |0\rangle) = (H \otimes I) |00\rangle$$

Producto de Kronecker (2)

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \otimes \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} a \begin{pmatrix} e & f \\ g & h \end{pmatrix} & b \begin{pmatrix} e & f \\ g & h \end{pmatrix} \\ c \begin{pmatrix} e & f \\ g & h \end{pmatrix} & d \begin{pmatrix} e & f \\ g & h \end{pmatrix} \end{pmatrix} = \begin{pmatrix} ae & af & be & bf \\ ag & ah & bg & bh \\ ce & cf & de & df \\ cg & ch & dg & dh \end{pmatrix}$$

$$A \in \mathbb{C}^{n \times m}$$

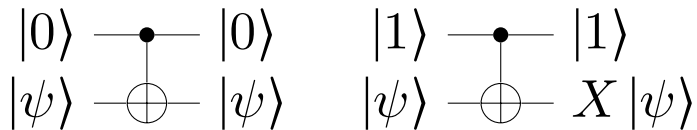
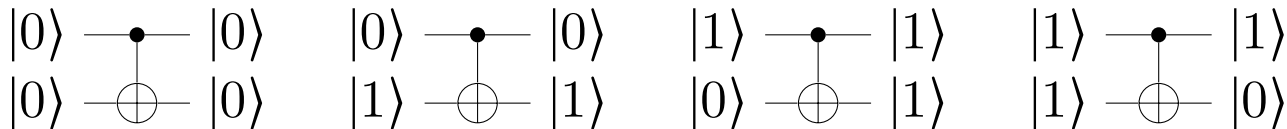
$$B \in \mathbb{C}^{n' \times m'}$$

$$A \otimes B \in \mathbb{C}^{nn' \times mm'}$$

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{aligned} H \otimes I &= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} & 1 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ 1 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} & -1 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{pmatrix} \\ &= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} \end{aligned}$$

Controlled-NOT (1)

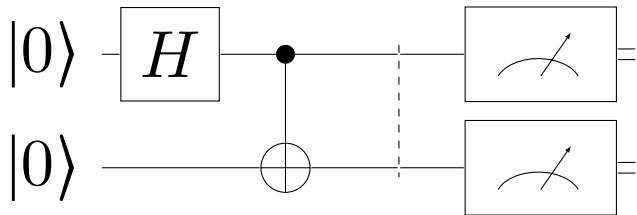


$$\begin{aligned}
 |00\rangle &\rightarrow \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} & |01\rangle &\rightarrow \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} & |10\rangle &\rightarrow \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} & |11\rangle &\rightarrow \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}
 \end{aligned}$$

Controlled-NOT (2)

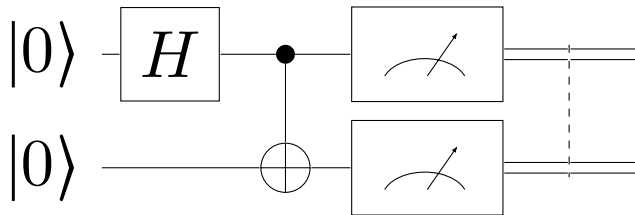
$$CNOT = CX = \left(\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array} \right) = \left(\begin{array}{c|c} I & 0 \\ \hline 0 & X \end{array} \right)$$

$$CX \begin{pmatrix} \alpha \\ \beta \\ \gamma \\ \delta \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \\ \delta \\ \gamma \end{pmatrix}$$



$$CX \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$CX(H \otimes I) |00\rangle$$

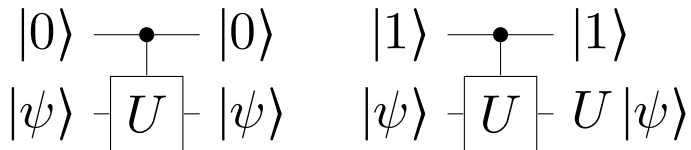


$$\mathcal{M} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 1/2 \\ 0 \\ 0 \\ 1/2 \end{pmatrix}$$

$$P(00) = 50\%$$

$$P(11) = 50\%$$

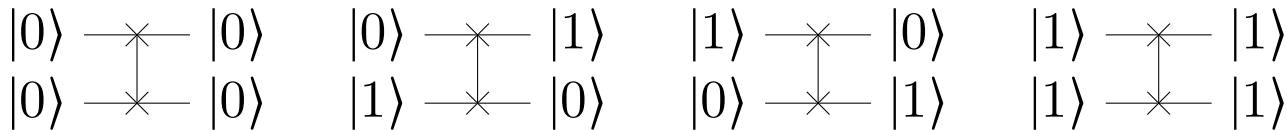
Controlled-U



$$CU = \left(\begin{array}{c|c} I & 0 \\ \hline 0 & U \end{array} \right)$$

$$U \simeq U' \not\Rightarrow CU \simeq CU'$$

Compuerta $SWAP$

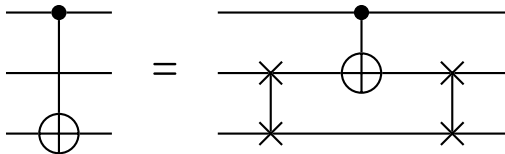
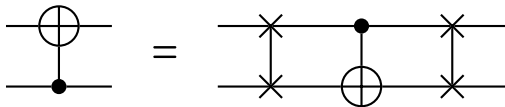


$$SWAP = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

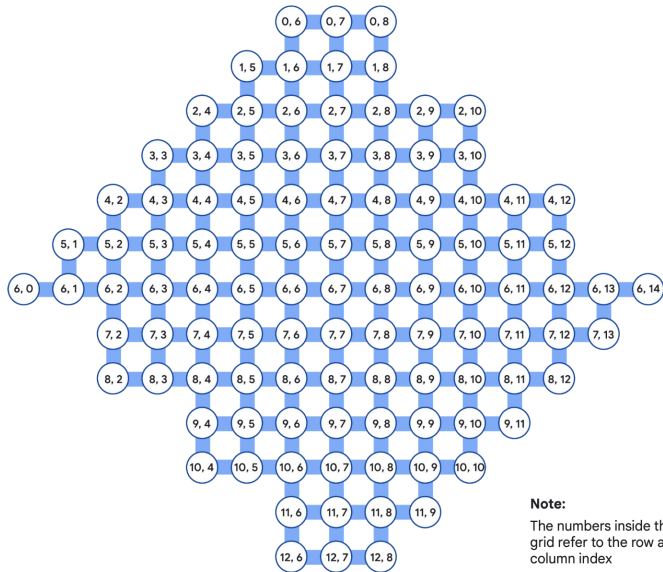
$$SWAP \begin{pmatrix} \alpha \\ \beta \\ \gamma \\ \delta \end{pmatrix} = \begin{pmatrix} \alpha \\ \gamma \\ \beta \\ \delta \end{pmatrix}$$

La compuerta $SWAP$ intercambia los qubits

Uso de la compuerta *SWAP*



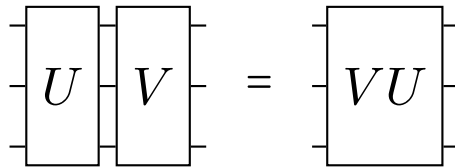
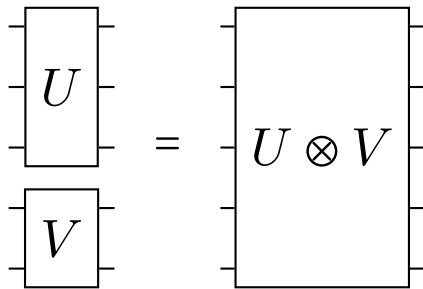
Superar las limitaciones de interconexión de los qubits



Note:

The numbers inside the qubit grid refer to the row and column index

De circuitos a matrices (1)



De circuitos a matrices (2)

